Ministero dell'Università e della Ricerca

FIS

POLITECNICO MILANO 1863 | DEPARTMENT OF ENERGY

IFP Energies nouvelles

Euler-Euler Simulation of Two-Phase Bubble Columns

22.09.2026 I Nicolò Varallo, PhD

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2. Numerical model
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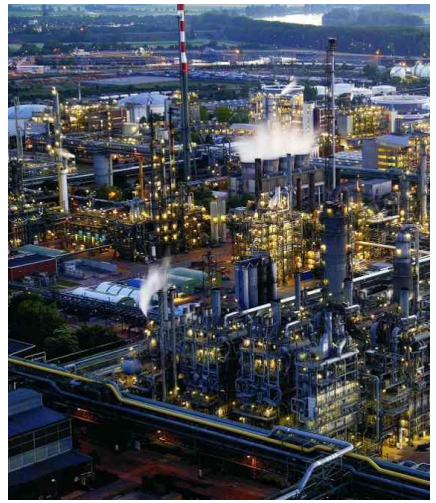
Introduction

Motivations and aims

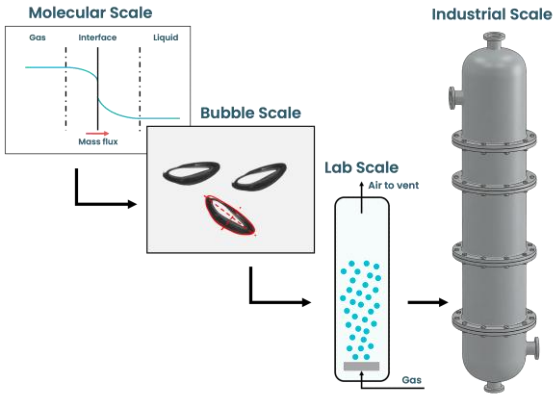


Large-scale gas–liquid processes are of great importance in many industrial applications:

- Chemical and biotechnology industries
- CO_2 absorption and carbon capture
- Pharmaceutical industry

Why bubble columns?

- Excellent heat and mass transfer performance
- Ease of operation
- Low operating and maintenance costs

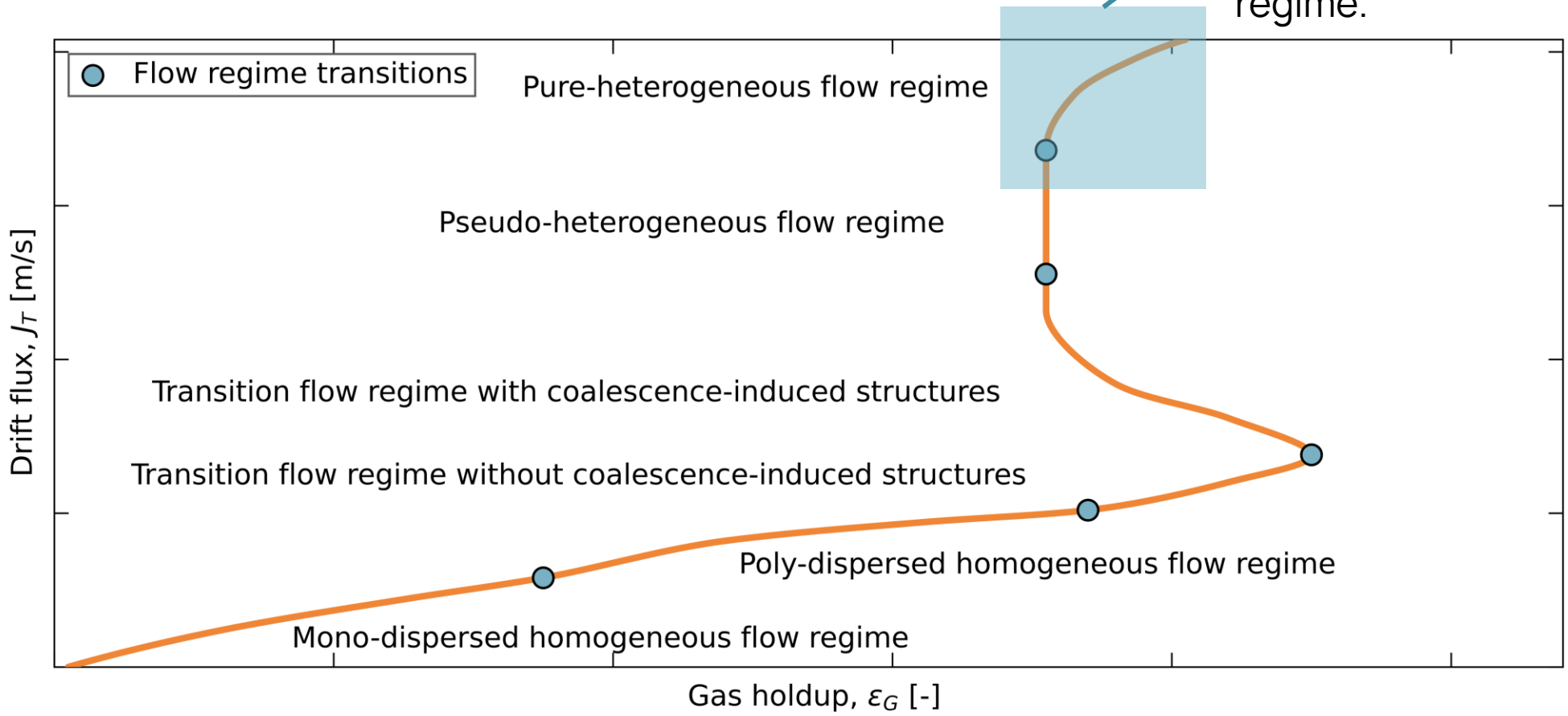


The performance of bubble columns is strongly influenced by their fluid dynamics, which are characterized by different **flow regimes**.

Introduction

Flow regimes

Industrial reactors are usually operated in the heterogeneous regime.



Besagni, G. (2021). Bubble column fluid dynamics: A novel perspective for flow regimes and comprehensive experimental investigations. *International Journal of Multiphase Flow*, *135*, 103510.

Numerical model

02

Numerical model

Governing equations

Mass conservation

$$\frac{\partial \alpha_k \rho_k}{\partial t} + \nabla \cdot (\alpha_k \rho_k \boldsymbol{u}_k) = 0$$

Momentum conservation

$$\frac{\partial}{\partial t}(\alpha_k \rho_k \, \boldsymbol{u}_k) + \nabla \cdot (\alpha_k \rho_k \, \boldsymbol{u}_k \boldsymbol{u}_k) = -\alpha_k \nabla p + \nabla \cdot (\alpha_k \bar{\boldsymbol{\tau}}_k) + \alpha_k \rho_k \boldsymbol{g} + \boldsymbol{M}_{I,k}$$

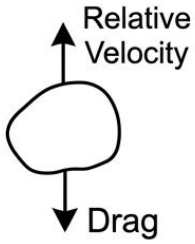
α_k - phase fraction, ρ_k - phase density, $\boldsymbol{u}_k$ - phase velocity, p – pressure, $\bar{\boldsymbol{\tau}}_k$ - strain rate tensor, $\mathbf{g}$ – gravitational acceleration, $\boldsymbol{M}_{I,k}$- interfacial momentum exchange

Numerical model

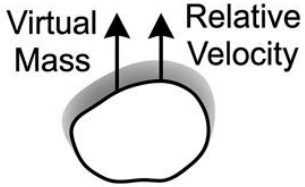
Bubble forces

$$\boldsymbol{M}_K = \boldsymbol{F}_D + \boldsymbol{F}_L + \boldsymbol{F}_{\mathrm{VM}} + \boldsymbol{F}_{\mathrm{WL}} + \boldsymbol{F}_{\mathrm{TD}}$$

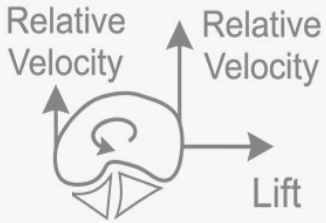
Drag



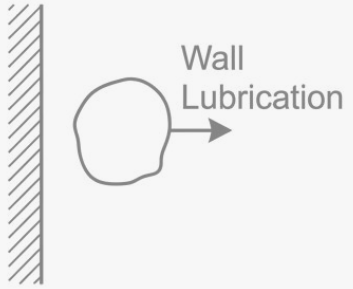
Virtual mass



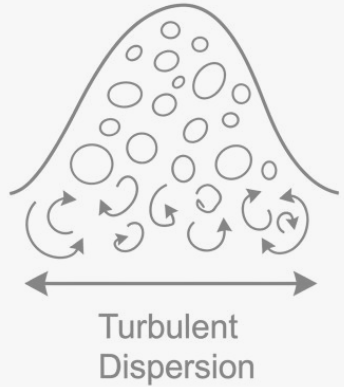
Lift



Wall lubrication

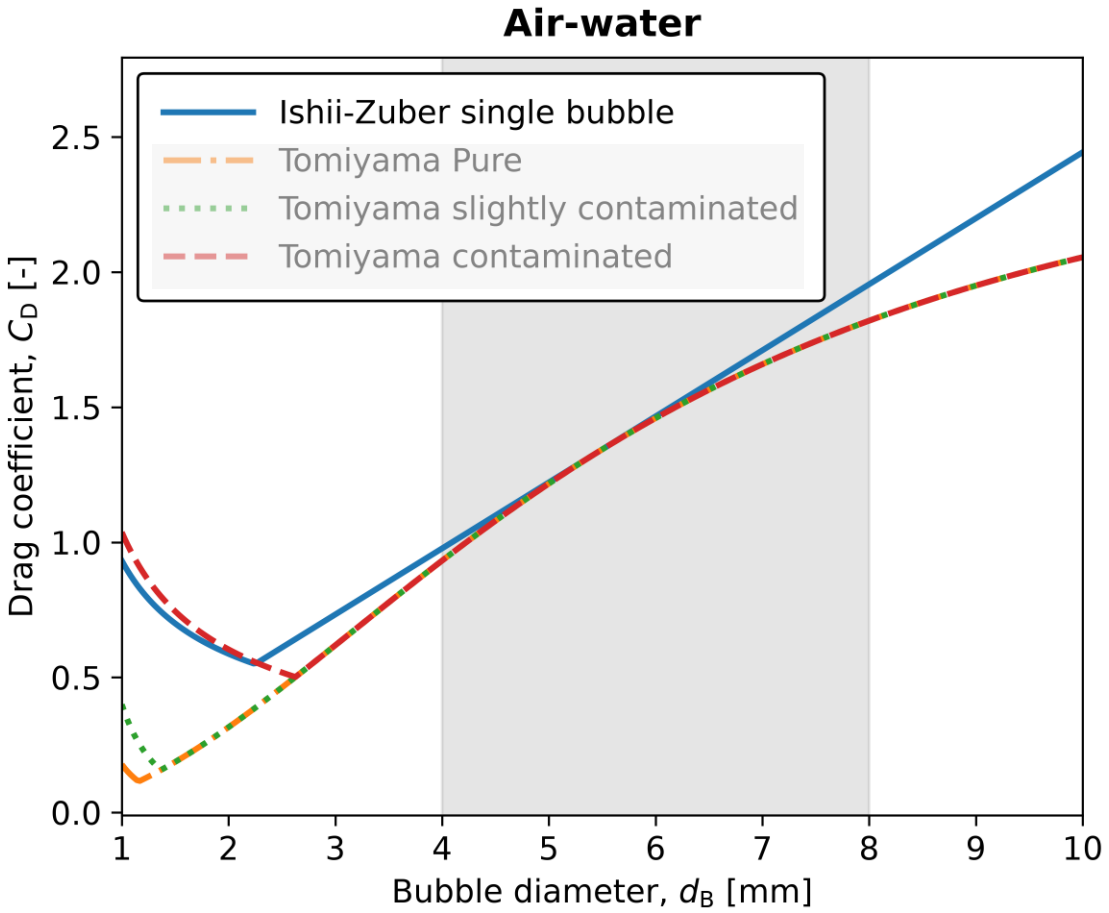


Turbulent dispersion



Numerical model

Bubble forces: drag



The single-bubble Ishii–Zuber formulation is used as the baseline, with swarm effects introduced separately.

$$C_D = \begin{cases} \dfrac{24}{Re_B}\left(1 + Re_B^{0.75}\right), & \dfrac{2}{3}\sqrt{Eo}\,Re_B < 24\left(1 + 0.1Re_B^{0.75}\right) \\ \min\left(\dfrac{2}{3}\sqrt{Eo}, \dfrac{8}{3}\right), & \dfrac{2}{3}\sqrt{Eo}\,Re_B < 24\left(1 + 0.1Re_B^{0.75}\right) \end{cases}$$

With

$$Re_B = \frac{\rho_L U_T d_B}{\mu_L}$$

$$Eo = \frac{(\rho_L - \rho_G) g d_B^2}{\mu_L}$$

Numerical model

Bubble forces: drag (i)

The drag models considered here are based on single-bubble formulations. To account for bubble swarm effects, a correction factor should be introduced to reduce the drag coefficient, particularly in the heterogeneous regime.

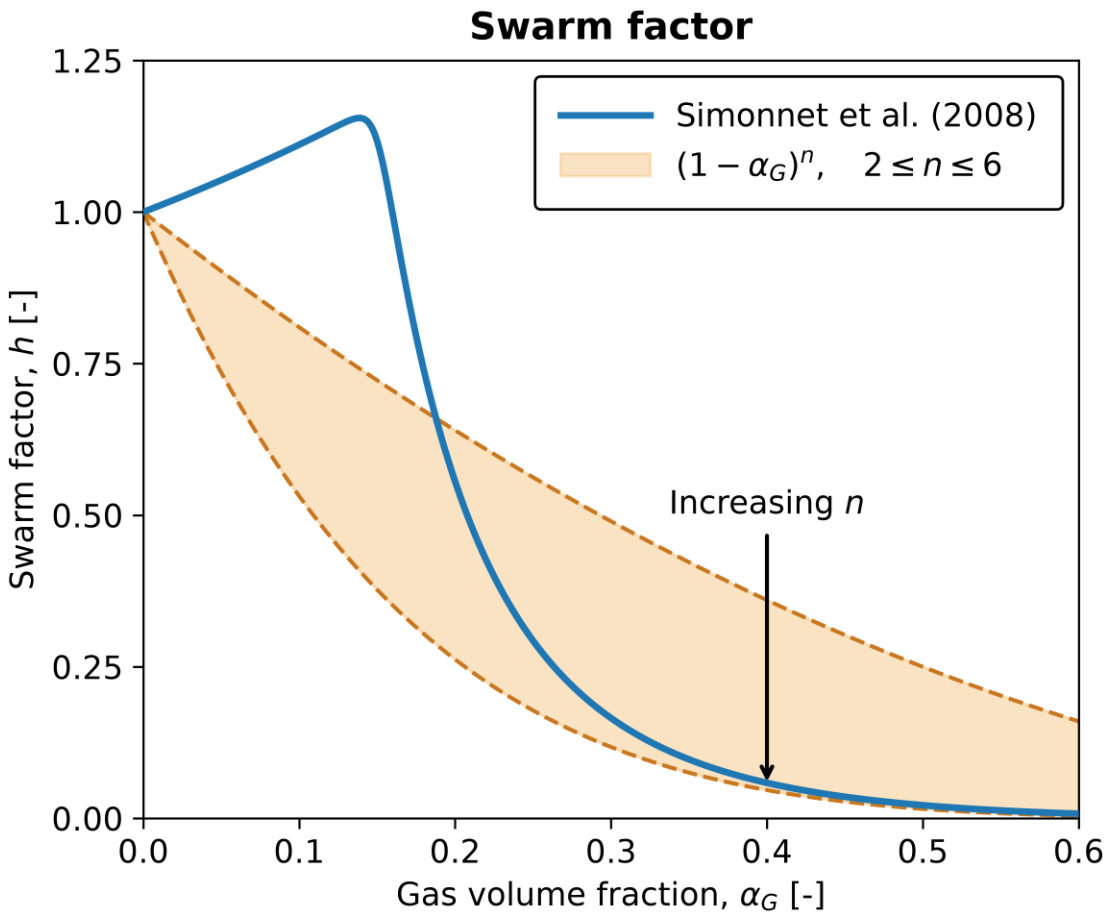
$$C_{\mathrm{D}} = h(\alpha_{\mathrm{G}}) C_{\mathrm{D,single\ bubble}}$$

$$h_{\mathrm{Simmonet}} = (1-\alpha_G)\left[(1-\alpha_G)^m + \left(4.8\frac{\alpha_G}{1-\alpha_G}\right)^m\right]^{-\frac{2}{25}}$$

✓ Experimentally based and simple to implement in CFD

✗ Limited validity range: $5 \leq d_{\mathrm{B}} \leq 10$ mm and $\alpha_{\mathrm{G}} < 0.30$

✗ The transition between drag enhancement and reduction may depend on the experimental setup



Numerical model

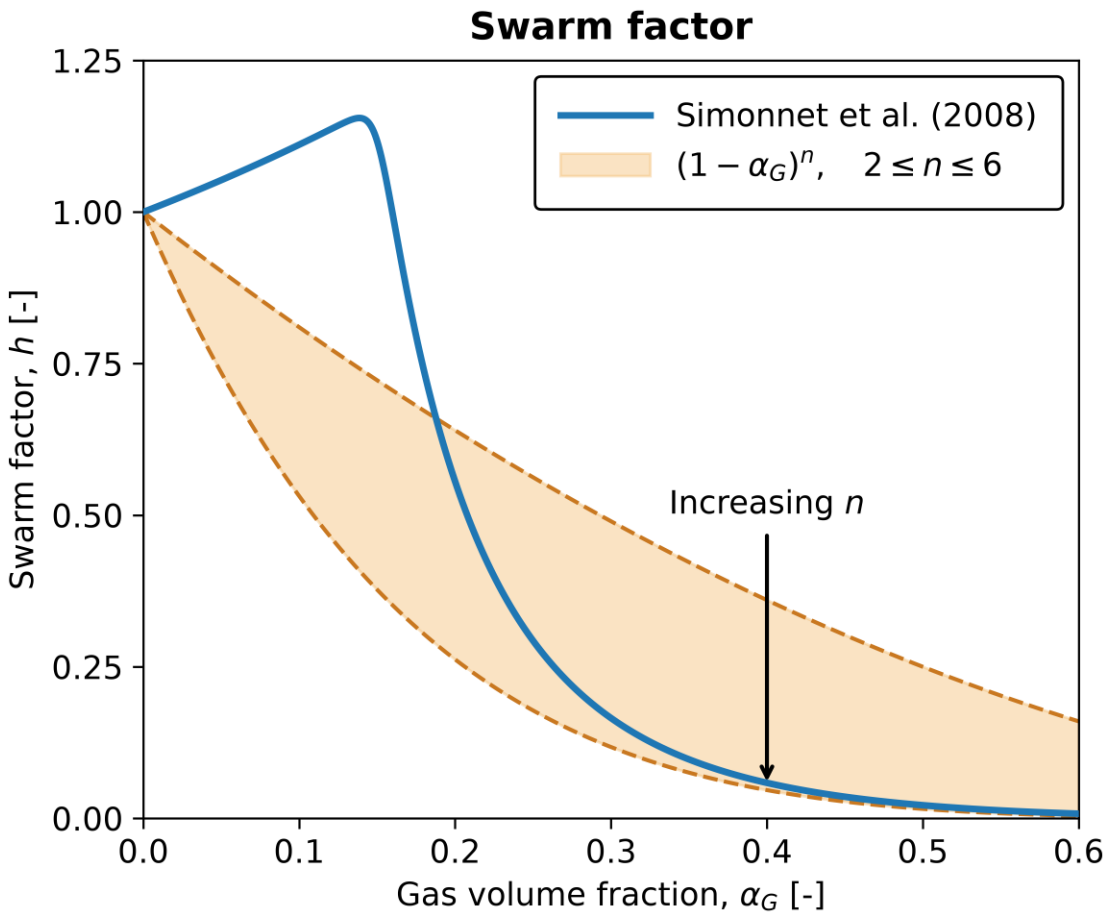
Bubble forces: drag (ii)

The drag models considered here are based on single-bubble formulations. To account for bubble swarm effects, a correction factor should be introduced to reduce the drag coefficient, particularly in the heterogeneous regime.

$$C_{\mathrm{D}} = h(\alpha_{\mathrm{G}}) C_{\mathrm{D,single\ bubble}}$$

$$h = (1 - \alpha_G)^n$$

The exponent (n) is determined through CFD simulations to match the experimental data in the heterogeneous regime.



Numerical model

Bubble forces: dispersion

- Dispersion force is essential for stable two-fluid simulations
- Ensures model hyperbolicity and grid-convergent solutions

Model formulation

$$\boldsymbol{F}_{\mathrm{DIS}} = A(\alpha_{\mathrm{G}})\frac{3}{4}C_{\mathrm{D}}\,\frac{\rho_{\mathrm{L}}}{\alpha_{\mathrm{L}}}C_{\mathrm{DIS}}f_{\Delta}f_{\alpha}|\boldsymbol{U}_{\mathrm{r}}|^{2}\sqrt{\alpha_{\mathrm{G}}\alpha_{\mathrm{L}}}\,\nabla\alpha_{\mathrm{G}}$$

Dispersion coefficient

$$C_{\mathrm{DIS}} = \sqrt{\frac{\alpha_{\mathrm{L}}\alpha_{\mathrm{G}}(\rho_{\mathrm{G}}\alpha_{\mathrm{L}} + \rho_{\mathrm{L}}C_{\mathrm{VM}})(\alpha_{\mathrm{L}} + C_{\mathrm{VM}})}{\rho_{\mathrm{L}}\alpha_{\mathrm{G}}\alpha_{\mathrm{L}} + \rho_{\mathrm{G}}\alpha_{\mathrm{L}}^{2} + \rho_{\mathrm{L}}C_{\mathrm{VM}}}}$$

Activation function

$$A(\alpha_{\mathrm{G}}) = H(\alpha_{G} - \alpha_{\mathrm{th}}) = \begin{cases} 1, & \alpha_{\mathrm{G}} > \alpha_{\mathrm{th}} \\ 0, & \text{otherwise} \end{cases}$$

Mesh correction factor

$$f_{\Delta} = 1 + C_{\Delta}\left[\max\left(\frac{d_{\mathrm{B}}}{\Delta} - 1{,}0\right)\right]^{n_{\Delta}}$$

$$\Delta = V_{\mathrm{cell}}^{1/3}$$

Void fraction correction factor

$$f_{\alpha} = 1 + C_{\alpha}\left[\max\left(\frac{\alpha_{\mathrm{G}} - \alpha_{\mathrm{th}}}{1 - \alpha_{\mathrm{th}}} - 0\right)\right]^{n_{\alpha}}$$

Panicker, N., Passalacqua, A., & Fox, R. O. (2018). On the hyperbolicity of the two-fluid model for gas–liquid bubbly flows. *Applied Mathematical Modelling*, *57*, 432-447.

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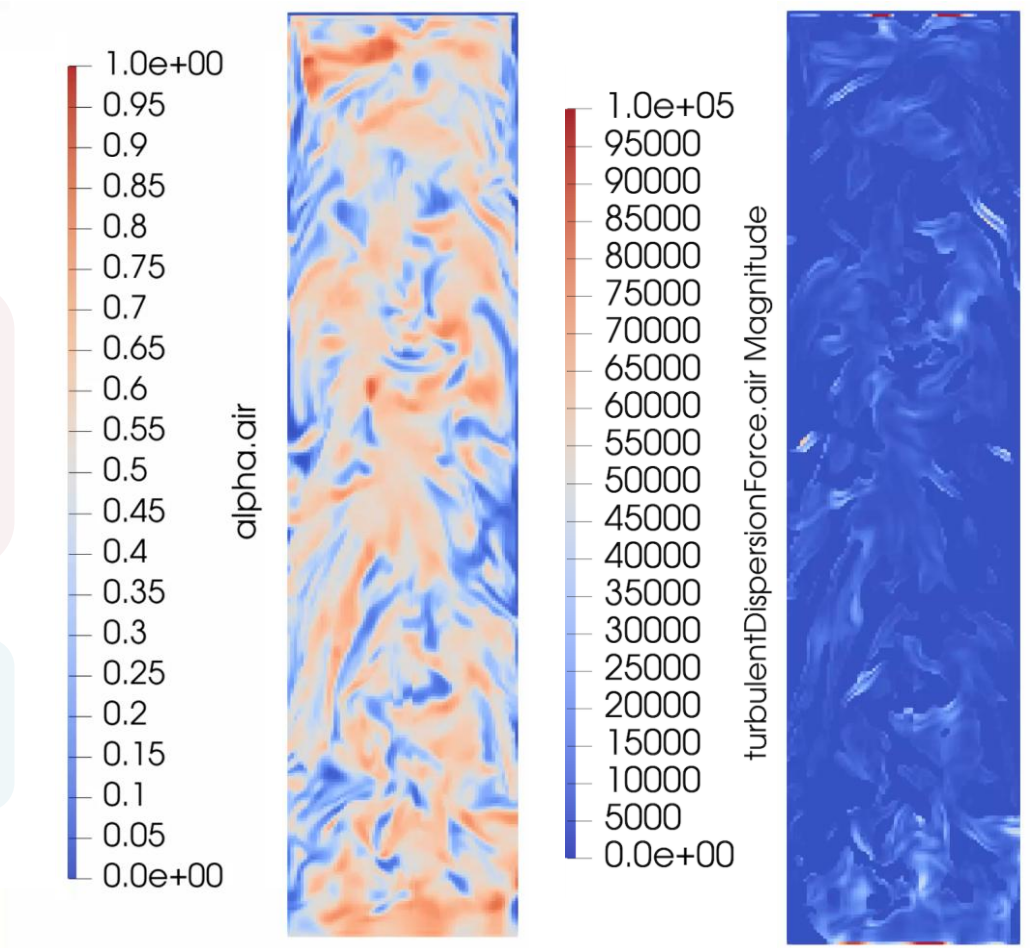
$\boldsymbol{U_{\mathrm{G}} = 0.35\ \mathrm{m/s};\ \sqrt{V_{\mathrm{cell}}}/d_{\mathrm{B}} = 0.75}$



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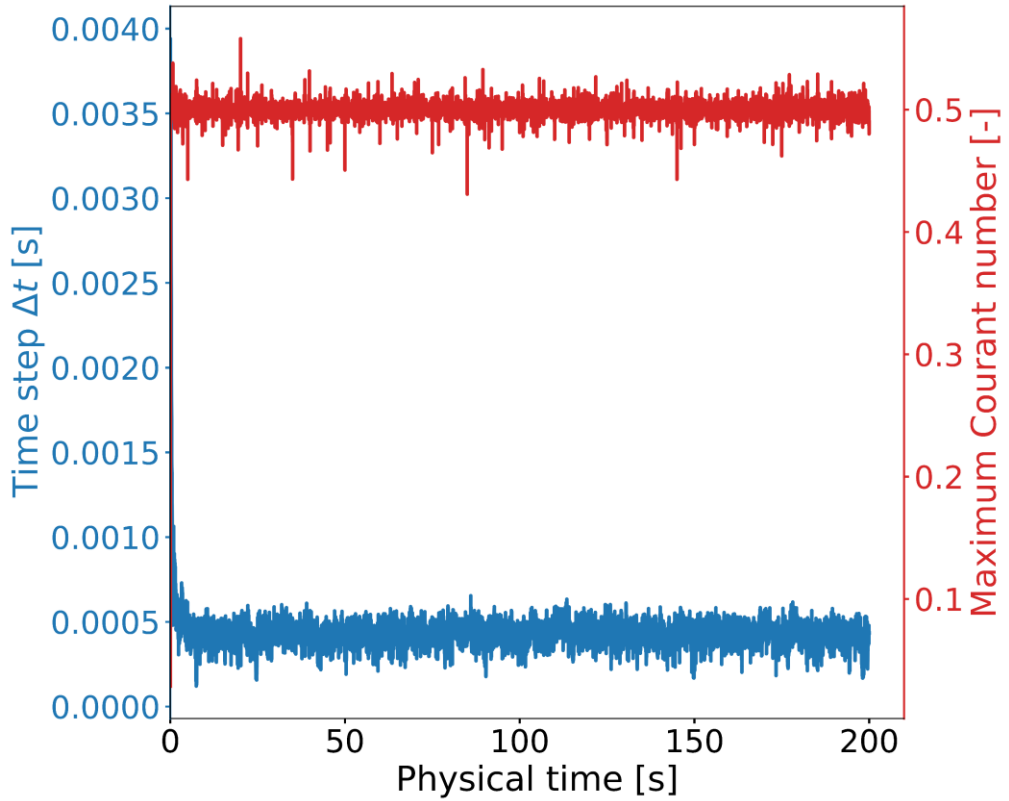

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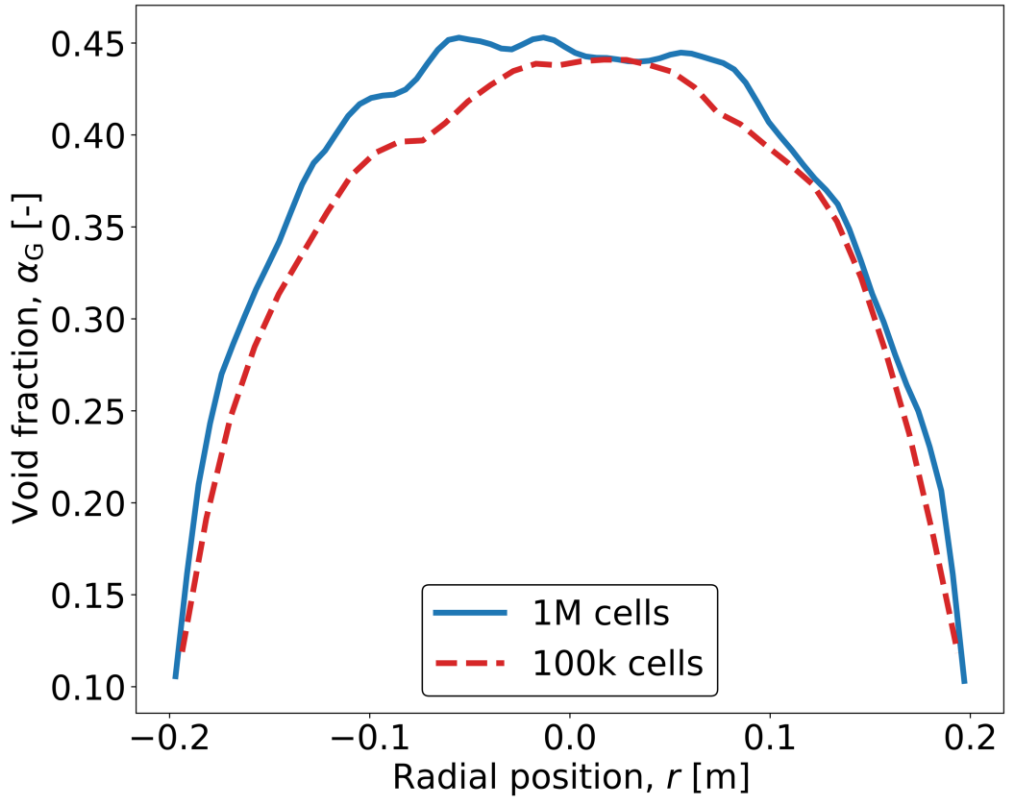


Panicker, N., Passalacqua, A., & Fox, R. O. (2018). On the hyperbolicity of the two-fluid model for gas–liquid bubbly flows. *Applied Mathematical Modelling*, *57*, 432-447.

Numerical model
Turbulence

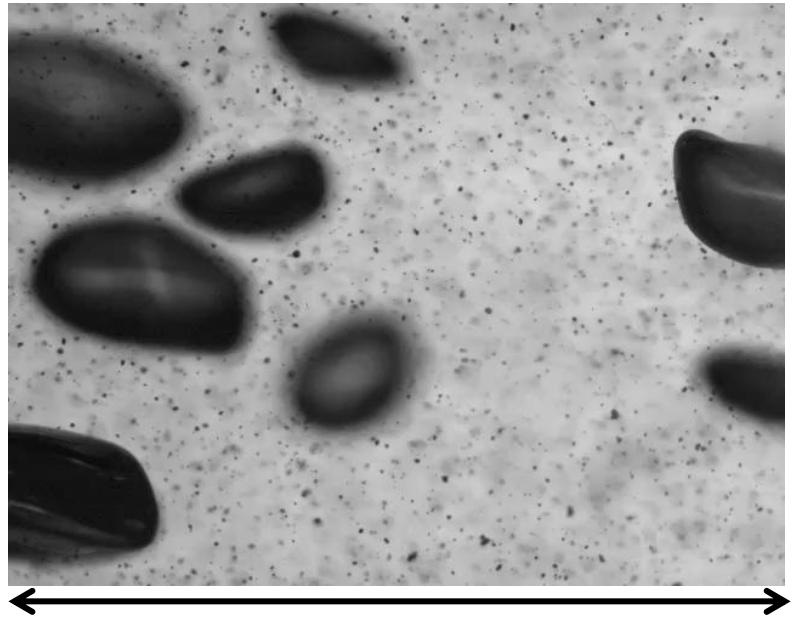
Turbulence is solved in the liquid only (k–ω SST); the gas phase enters as extra source terms in the k and ω equations.

$$S_{k^{\mathrm{BIT}}} = \boldsymbol{F}_D \cdot (\boldsymbol{u}_{\mathrm{G}} - \boldsymbol{u}_{\mathrm{L}})$$

$$S_{\varepsilon^{\mathrm{BIT}}} = C_{\varepsilon \mathrm{B}} \frac{S_{k^{\mathrm{BIT}}}}{\tau}, \quad \tau = \frac{d_{\mathrm{B}}}{\sqrt{k}}$$

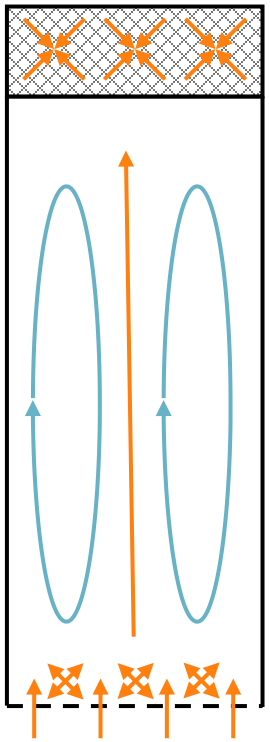
$$S_{\omega^{\mathrm{BIT}}} = \frac{1}{C_\mu k} S_{\varepsilon^{\mathrm{BIT}}} - \omega \frac{S_{k^{\mathrm{BIT}}}}{k}$$

$C_{\varepsilon \mathrm{B}} = 1.0$. Assumes complete conversion of the drag work — calibrated on air–water.



Numerical model

Boundary conditions and mesh



Volume at the outlet

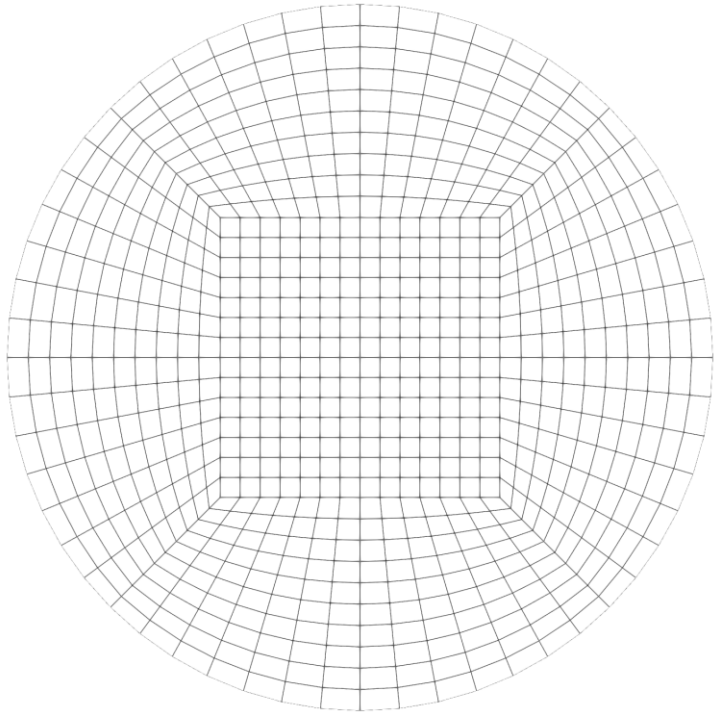
Inlet boundary condition: fixed values of α_G, v_G, k_L and ω_L

Sink

Source

Continuous water

Dispersed air

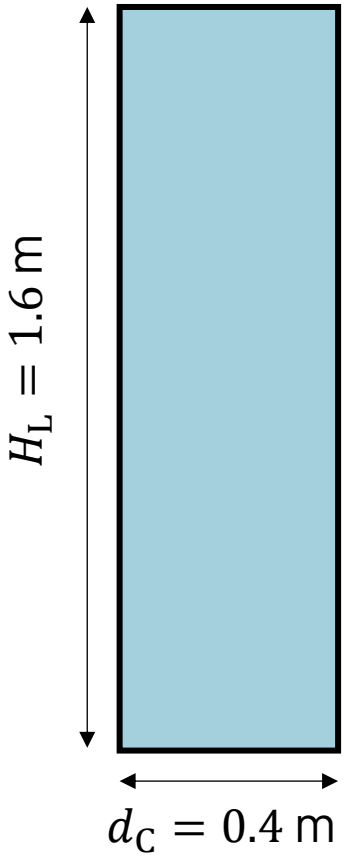


- Hexahedral mesh
- $\frac{L_{mesh}}{d_B} \approx 1.5$

Validation benchmark

03

Validation benchmark



- Reference: Gemello (2018) [1]
- $U_G = 1 - 35$ cm/s
- Gas phase: air
- Liquid phase: **water, water-Ethanol 0.01%, water-Ethanol 0.05%**

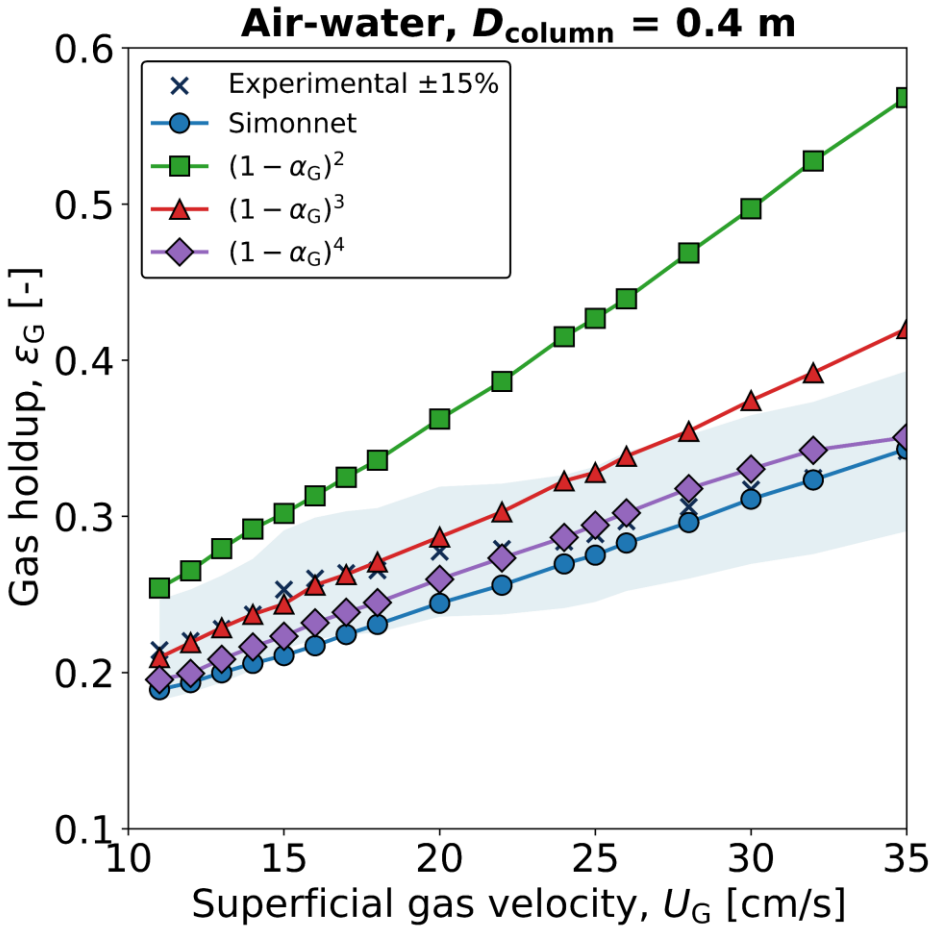
[1] Gemello, L., 2018. *Modelling of the hydrodynamics of bubble columns using a two-fluid model coupled with a population balance approach*. PhD Thesis, Université de Lyon – Politecnico di Torino.

Results

04

Results

Swarm-factor

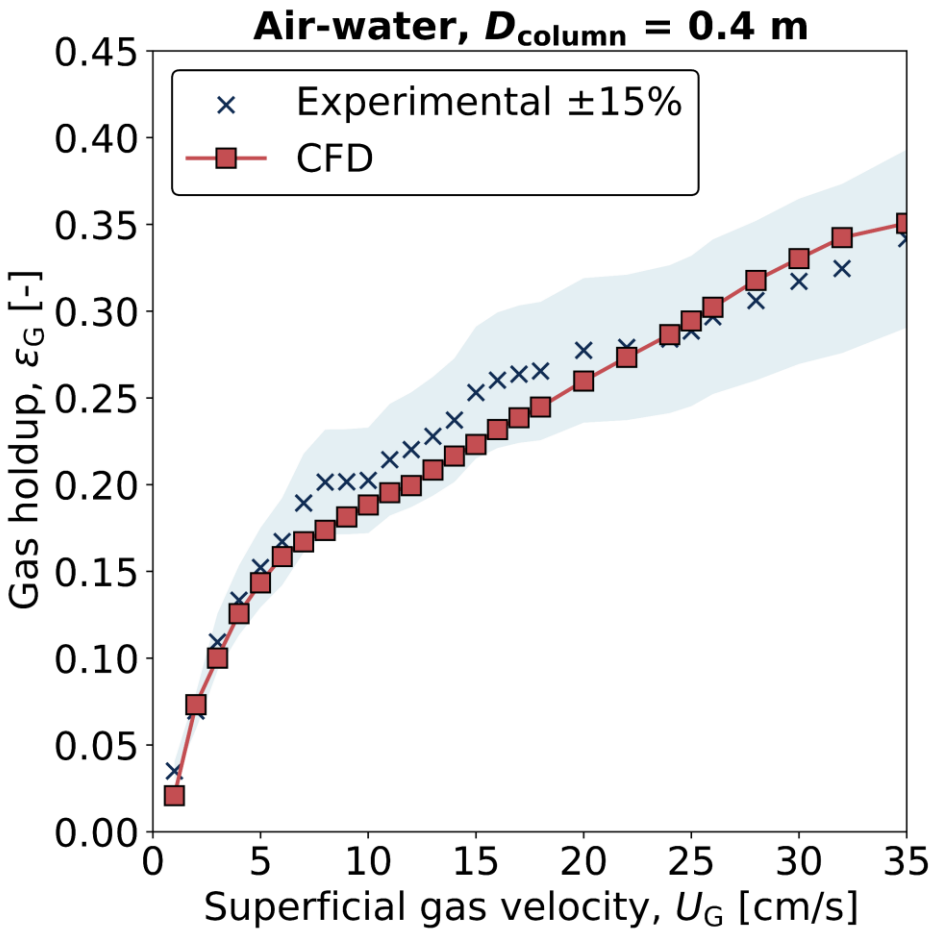


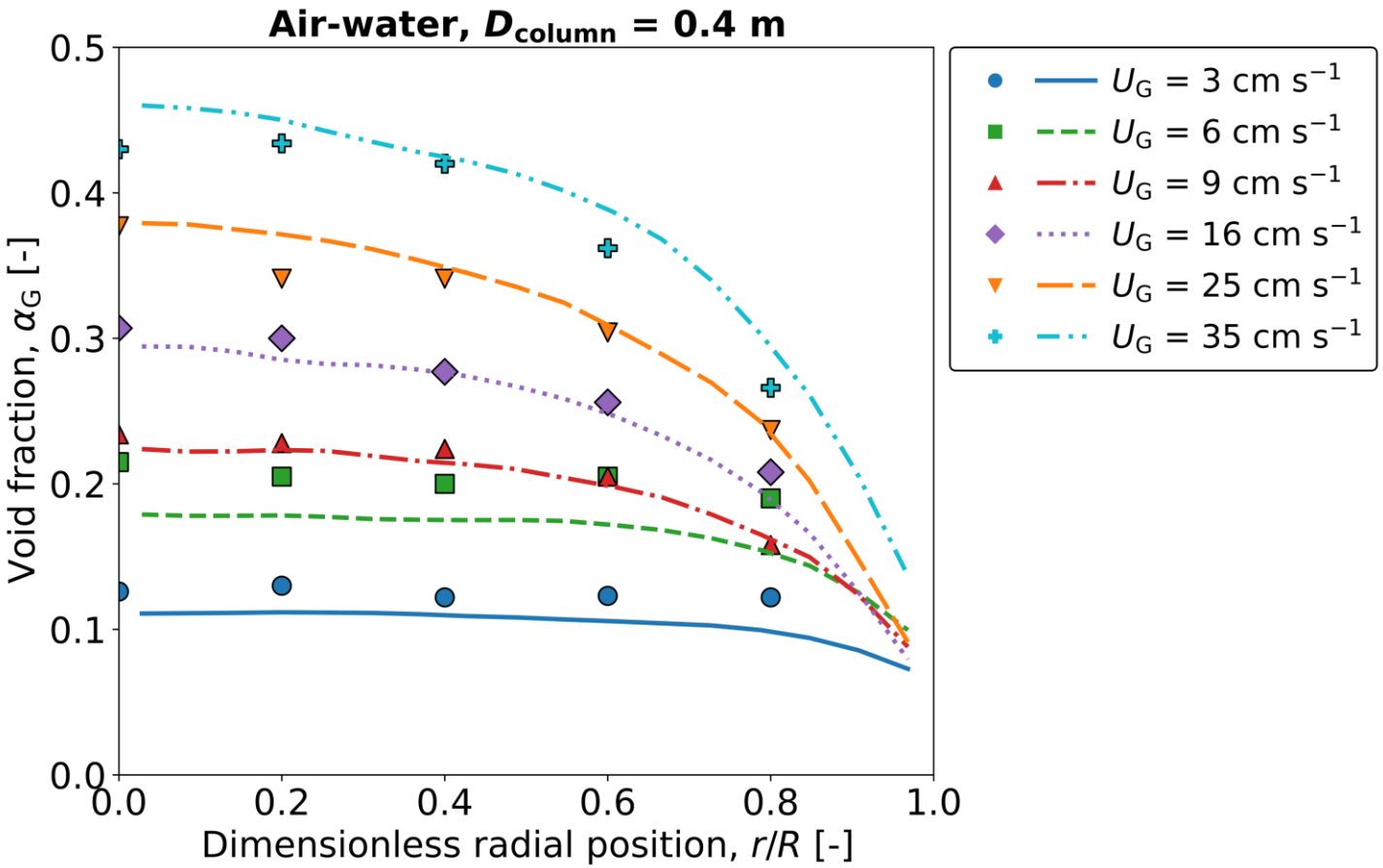
- Simonnet model tends to underpredict the experimental gas holdup
- Increasing the exponent in the swarm correction progressively decreases the predicted gas holdup
- $(1-\alpha_G)^4$shows the best agreement among the tested power-law swarm corrections

⟹ **The $(1-\alpha_G)^4$swarm correction was therefore adopted for all subsequent simulations**

Results

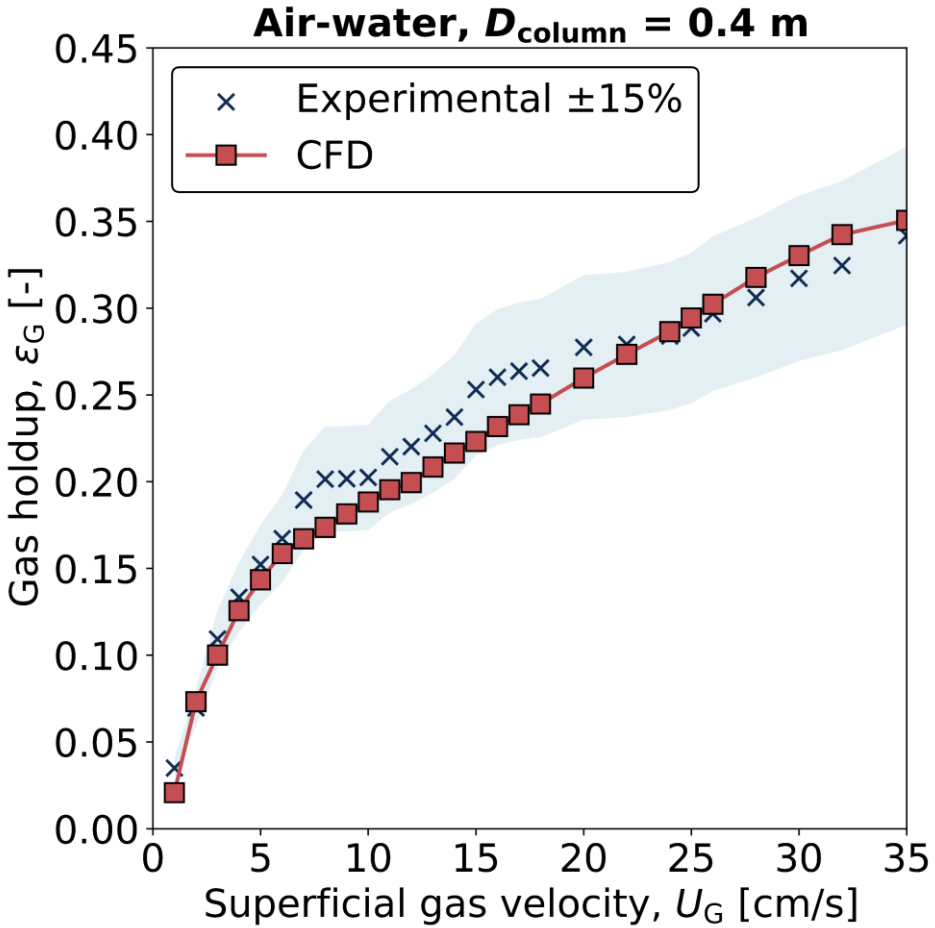
Air-water

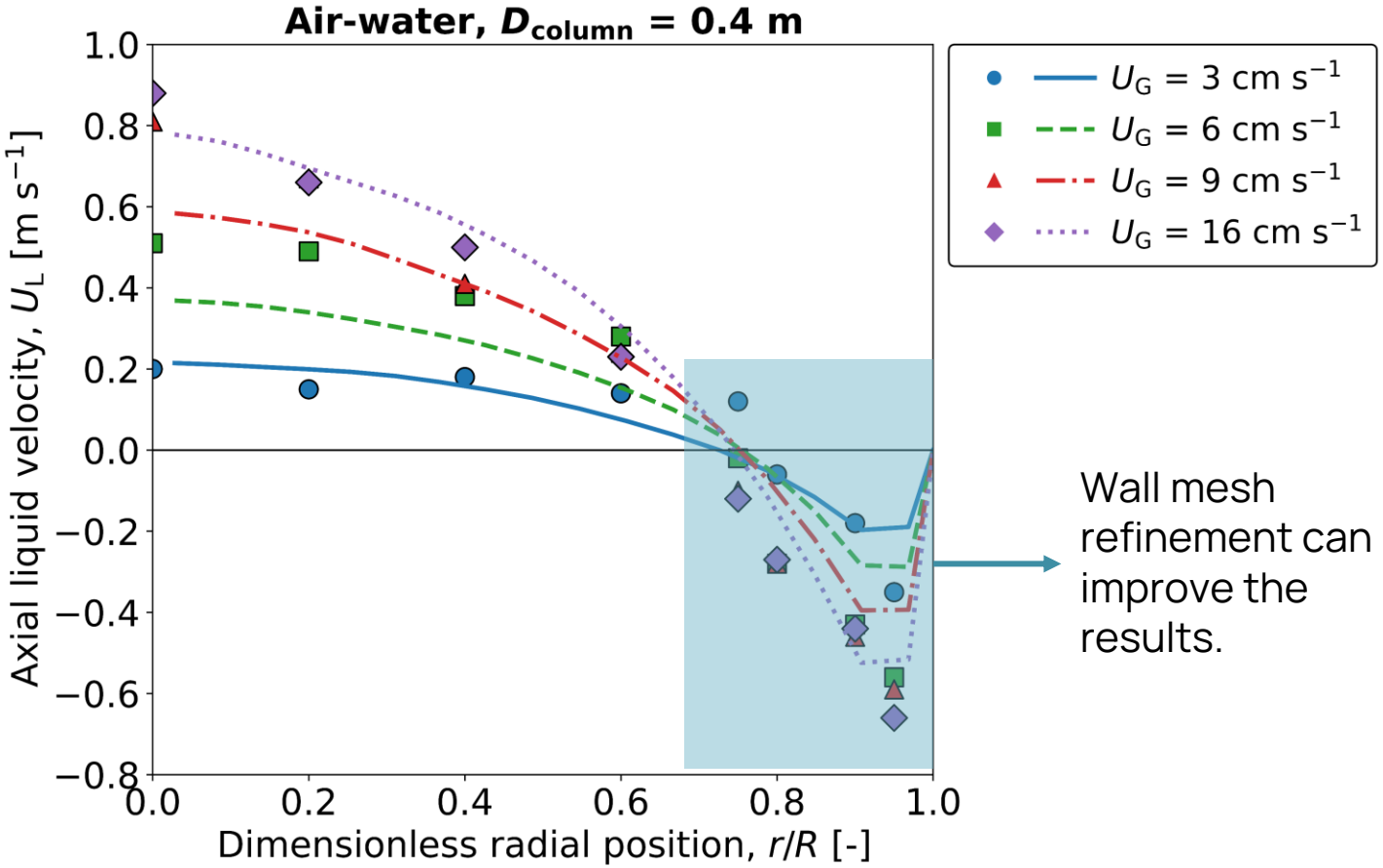




Results

Air-water

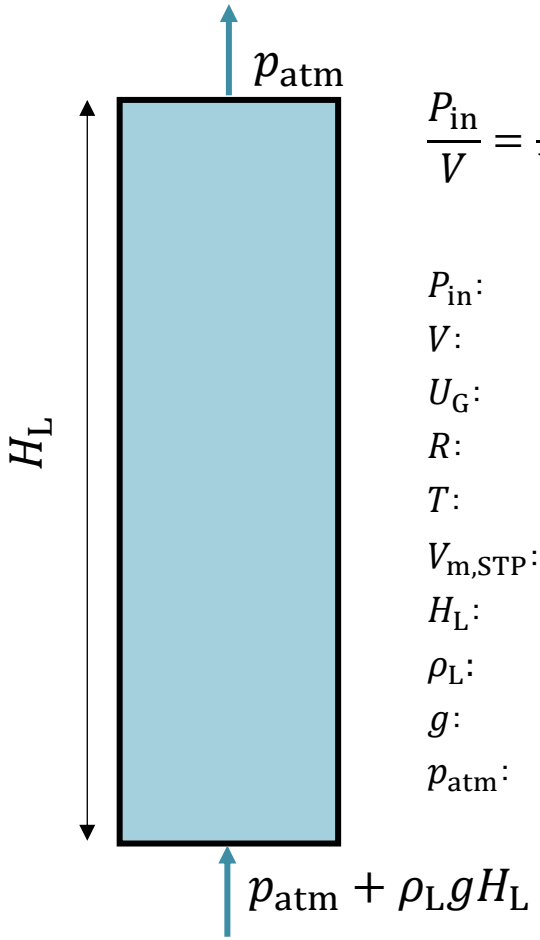




Wall mesh refinement can improve the results.

Results

Why do we need bubble induced turbulence?



$$\frac{P_{\mathrm{in}}}{V} = \frac{U_{\mathrm{G}} RT}{V_{\mathrm{m,STP}} H_{\mathrm{L}}} \ln\left(1 + \frac{\rho_{\mathrm{L}} g H_{\mathrm{L}}}{p_{\mathrm{atm}}}\right) \approx \frac{1}{V} \rho_{\mathrm{L}} \int_V \alpha_{\mathrm{L}} C_\mu k_{\mathrm{L}} \omega_{\mathrm{L}}$$

P_{in}: minimum gas power input [W]
V: column volume [m³]
U_{G}: superficial gas velocity at STP
R: universal gas constant [8.314 J/(mol·K)]
T: absolute temperature [K]
$V_{\mathrm{m,STP}}$: molar gas volume at STP [22.4 × 10⁻³ m³/mol]
H_{L}: ungassed liquid height [m]
ρ_{L}: liquid density [kg/m³]
g: gravitational acceleration [9.81 m/s²]
p_{atm}: gas pressure at column outlet [Pa]

$U_{\mathrm{G}} = 3$ cm/s

$$\begin{cases} \dfrac{P_{\mathrm{in}}}{V} = 278.95 \dfrac{\mathrm{W}}{\mathrm{m}^3} \\ \dfrac{1}{V} \rho_{\mathrm{L}} \displaystyle\int_V \alpha_{\mathrm{L}} C_\mu k_{\mathrm{L}} \omega_{\mathrm{L}} \Big|_{\text{NO BIT}} = 18.35 \dfrac{\mathrm{W}}{\mathrm{m}^3} \\ \dfrac{1}{V} \rho_{\mathrm{L}} \displaystyle\int_V \alpha_{\mathrm{L}} C_\mu k_{\mathrm{L}} \omega_{\mathrm{L}} \Big|_{\text{BIT}} = 242.97 \dfrac{\mathrm{W}}{\mathrm{m}^3} \end{cases}$$

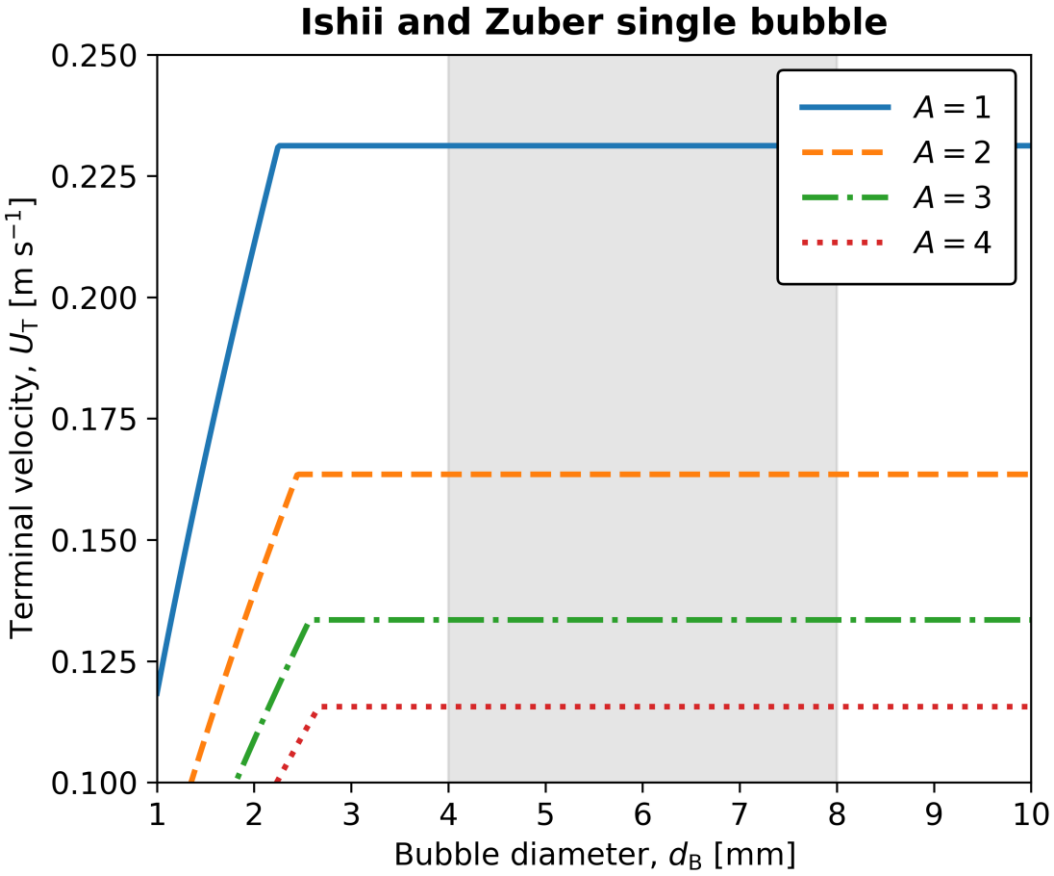
Roels, J. A., & Heijnen, J. J. (1980). Power dissipation and heat production in bubble columns: Approach based on nonequilibrium thermodynamics. Biotechnology and Bioengineering, 22(11), 2399-2404.

Results

Different liquid phases

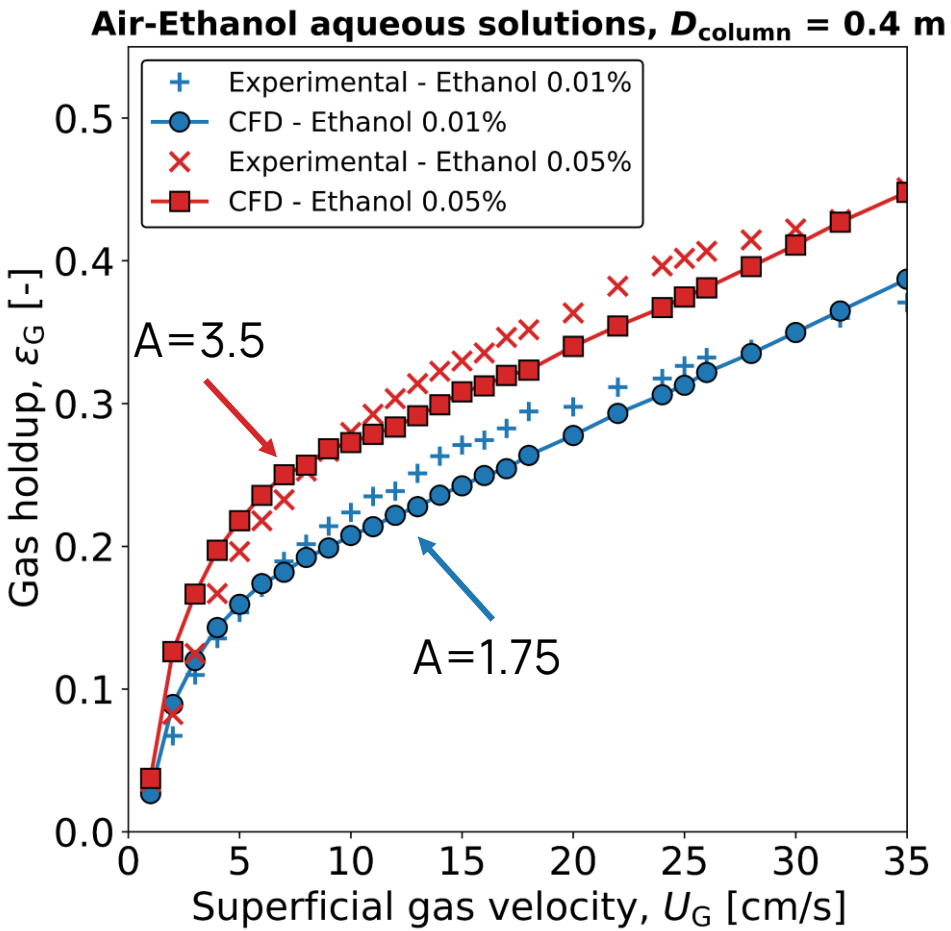
- Drag correlations work well for air–water systems but tend to overpredict bubble rise velocity for fluids with properties different from those of water
- A correction factor is introduced into the single-bubble drag coefficient to account for the effects of liquid viscosity and surface tension
- The correction factor is calibrated by matching the CFD-predicted gas holdup to the experimental data

$$C_{\mathrm{D}} = A C_{\mathrm{D,air-water}}$$



Results

Different liquid phases



	A [-]	U_T^* [cm/s]
Water	1	0.23
Ethanol 0.01 %	1.75	0.17
Ethanol 0.05 %	3.5	0.12

*Evaluated for $d_B = 5$ mm

Conclusions

- **Bubble swarm effects must be accounted for** to correctly predict gas holdup in the heterogeneous regime
- The **power-law swarm correction** provides good agreement with experimental air–water data
- For **liquids with different properties**, an additional drag correction is required to account for viscosity and surface tension effects
- The results emphasize the importance of developing **closure models applicable beyond air–water systems**.

Not only bubbly flows !

Euler-Euler simulation of parallel risers



Schlegel, Fabian, Bilde, Kasper Gram, Draw, Mazen, Evdokimov, Ilya, Hänsch, Susann, Kamble, Vikrant Vinayak, ... Varallo, Nicolo. (2026, June 29). Multiphase Code Repository by HZDR for OpenFOAM Foundation Software (Version 13-s.1-hzdr.2). Rodare. http://doi.org/10.14278/rodare.4741

Thank you for the attention !

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Contact

nicolo.varallo@polimi.it

www.auriga.polimi.it

www.polimi.it